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1 a i) Solve each of the following systems: $A\,X_1 = B_1$ and $A\,X_2 = B_2$, where

$$A = \begin{bmatrix} 6 & 4 & 3 \\ 20 & 15 & 12 \\ 15 & 12 & 10 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} 6 & -4 & 3 \\ -20 & 15 & -12 \\ 15 & -12 & 10 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 13 \\ 47 \\ 37 \end{bmatrix}, B_2 = \begin{bmatrix} 13.1 \\ 46.9 \\ 37.1 \end{bmatrix}$$

ii) Comment on the results and compute the condition number of A.

iii) Write a MATLAB commands to solve the system $A\,X_2 = B_2$ 7 M

b Use the Newton's method to find the negative zero of the function:

$$f(x) = e^x - tan^{-1}x - 1.5.$$

i) Start at $x_0 = -12$ to get the solution accurate to 6 decimal places,

ii) write a MATLAB command to find this root. 7 M

c Carry out two iterations of Newton's method on the following system. Use starting values $x_0 = -1$ $y_0 = 4$.

$$x^2 - y^2 + e^x cosy = -1$$
$$2xy + e^x siny = 0.0$$

7 M

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2 a Given the system of linear differential equations

$$y_1' = 3y_1 + y_2$$
$$y_2' = 2y_1 + 2y_2$$

Put the system in the matrix form $y' = Ay$. Determine the eigenvalues and eigenvectors of A then solve the system. 7 M

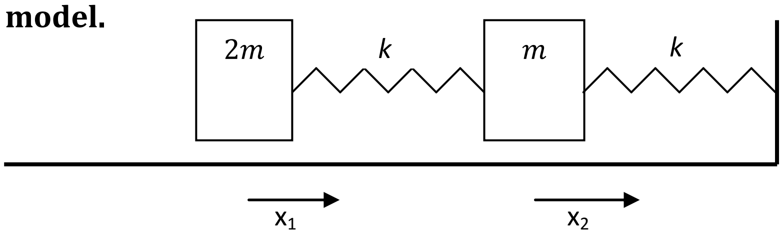
b Consider the spring-mass system as shown in the figure.

Assume the two mass-displacements to be denoted by x_1, x_2 and let each spring has the same spring constant $k = 20$ and let $m = 10$, then:

i) Applying equilibrium equations, write the equations of motion for free vibration.

ii) Apply $x_i = A_i \sin(\omega t)$ to formulate the problem as an eigenvalue problem.

iii) Write a MATLAB program to calculate the eigenvalues and eigenvectors of the model.



7 M

3 a Given the initial value problem $y' = \frac{x}{y}, \quad y(0) = 1,$ 8 M

i) Solve the problem by Runge Kutta method using $h = 0.1$ from $x = 0$ *to* $x = 0.3$.

ii) Write a MATLAB code that uses *h=0.05* to solve the problem from $x = 0$ *to* $x = 10$.

b Solve $\frac{dy}{dx} = z, \quad \frac{dz}{dx} = 6y - z, \quad y(0) = 3,$

$z(0) = 1, \quad h = 0.5$ from $x = 0$ to $x = 1$. 7 M

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4 a Solve Poisson's equation $\nabla^2 u = 4y$ on a thin plate of dimension 1.5×2 units,

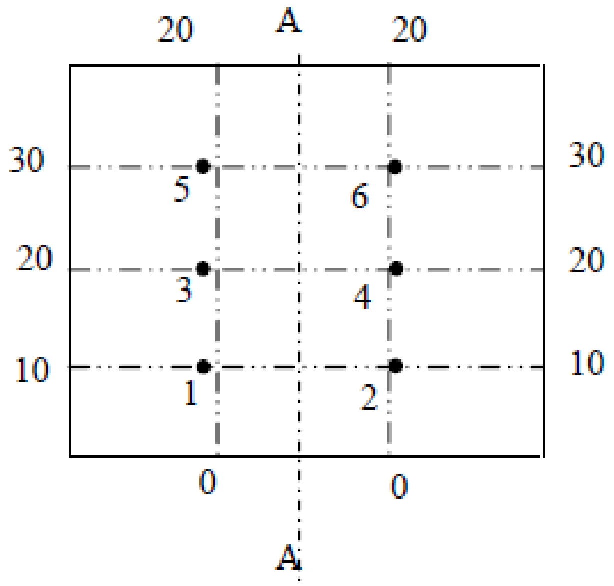
$u(0, y) = 20y,$

$u(1.5, y) = 20y,$

$u(x, 0) = 0,$

$u(x, 2) = 30 + 20x(x - 1.5),$

with $h = k = 0.5$



10 M

b Solve the wave equation of the vibrating string, $u_{tt} = u_{xx}$ if $0 \le x \le 1$ with $h = k = 0.2$, starting from its equilibrium position $(u(x,t) = f(x) = 0)$, and with initial velocity $u_t(x, 0) = g(x) = \sin \pi x$.

Find its displacement at time t = 0.4 and x = 0.2, 0.4, 0.6, 0.8 10 M

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